Experiments with Rotating Fluids.

By G. I. Taylor, F.R.S.

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[Plate 2.]

It is well known that predictions about fluid motion based on the classic hydrodynamical theory are seldom verified in experiments performed with actual fluids. The explanation of this want of agreement between theory and experiment is to be found chiefly in the conditions at the surfaces of the solid boundaries of the fluid.

The classical hydrodynamical theory assumes that perfect slipping takes place, whereas in actual fluids the surface layers of the fluid are churned up into eddies. In the case of motions which depend on the conditions at the surface, therefore, no agreement is to be expected between theory and experiment. This class of fluid motion, unfortunately, includes all cases where a solid moves through a fluid which is otherwise at rest.

On the other hand, there are types of fluid motion which only depend to a secondary extent on the slip at the boundaries. For this reason theoretical predictions about waves and tides, or about the motion of vortex rings, are in much better agreement with observation than predictions about the motion of solids in fluids. Some time ago the present writer* made certain predictions about the motion of solids in rotating fluids, or rather about the differences which might be expected between the motion of solids in a rotating fluid and those in a fluid at rest. These predicted features of the motion did not depend on conditions at the boundaries. It was therefore to be anticipated that they might be verified by experiment. The experiments were carried out and the predictions were completely verified.

In view of the interest which attaches to any experimental verification of theoretical results in hydrodynamics, and more particularly to verifications of those concerning the motion of solids in fluids, it seems worth while to publish photographs showing the experiments in progress. In the second and third part of the paper further experiments are described in which theoretical predictions are verified in experiments with water.

Motion of Cylinder and Sphere in Rotating Fluid.

In these experiments a solid cylinder and a solid sphere, of the same density as water, were drawn through a rotating vessel containing water.

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* 'Roy. Soc. Proc.,' A, vol. 93, p. 99 (1917).

The threads by means of which these solids were dragged, passed through small rings attached to the vertical wall of the circular rotating glass vessel in which the water was contained. The solids were initially attached to the opposite point of this vessel to that at which the rings were attached. Under these circumstances, if the vessel were not rotating, a symmetrical body like a sphere or cylinder would evidently pass along a diameter through the centre of the apparatus when towed by the threads.

When the vessel was rotating, however, the theoretical prediction* was that the cylinder would pass through the centre of the apparatus just as if the whole system were not rotating, while a sphere, or any symmetrical three-dimensional body, would be deflected and would pass to one side of the centre. The verification of this prediction was first made in an apparatus with which it was difficult to obtain photographs owing to the difficulty of throwing a light through the water towards a camera placed on the axis of rotation above the apparatus. A new apparatus was therefore devised, in which the vertical central spindle used in the previous apparatus was done away with.

Two dishes in the form of circular cylinders were made. Each had a thin plate-glass bottom, about 1 mm. thick. The diameter of one of them was made about 1/8 inch larger than the other, so that the smaller one would fit inside the larger one, leaving a space of about 1/16 inch all round. The inner dish was filled about two-thirds full of water, and the outer one was filled quite full. The inner cylinder would then float in the outer one. It was driven round a vertical axis by means of a jet of water projected at its outside surface. When the apparatus was set up so as to run truly, it was found that a very uniform rate of rotation would be obtained by this method. The whole apparatus stood in a trough with a plate glass bottom, in which the water which overflowed could be collected. This apparatus is shown in text-fig. 1.

In order to take instantaneous photographs, the apparatus was illuminated from underneath, and a camera with a lens of 15 inches focus was fixed about 6 feet above it. Two methods of illumination were used. In the first method the direct light from a spark between an aluminium wire and a hole in an aluminium plate was focussed, by means of a condenser, on the lens of the camera. In the second method use was made of diffused light, a mercury vapour spark being used to illuminate a ground-glass plate placed close under the apparatus. Plate 2, figs. 1, 2, 5 and 6, were taken by the first method, while figs. 3 and 4 were taken by the second. It is worth pointing out that the apparatus is very simple, and that it is easy to project all the experiments to be described in this paper on to a screen by means of a lantern.

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* *Loc. cit.*, p. 105.

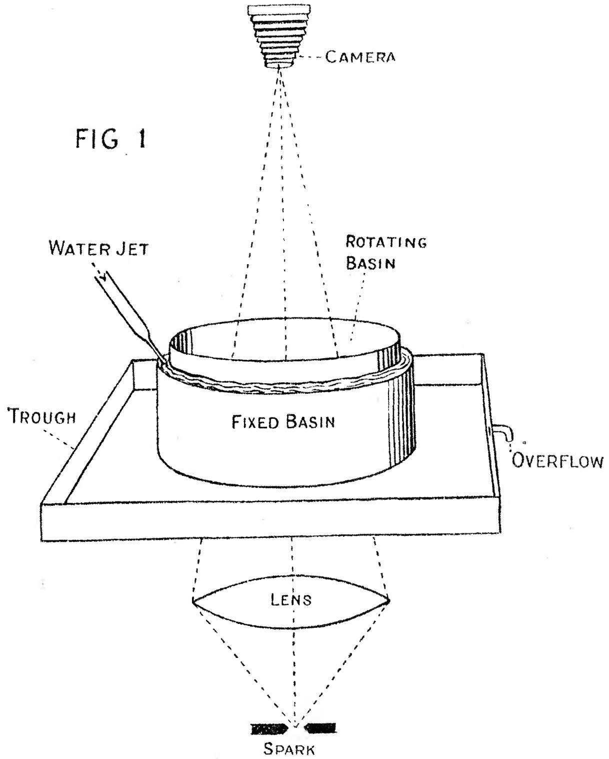

Fig. 1.—General Arrangement of Apparatus.

It was thought at first that it might be difficult to arrange to tow the solids through the rotating basin in such a way that no appreciable variation in the speed of rotation would occur. To avoid this difficulty, the threads used to tow the solids were led from the edge of the apparatus back to the centre, and were pulled upwards from that point. The method adopted for doing this was to fix a transparent celluloid bridge across the basin in such a position that its centre line ran from the initial position of the solid to the point towards which the threads were towing it. This transparent bridge shows up clearly as a broad band in figs. 1 and 2 (Plate 2). The threads passed through a small hole in the centre of the bridge, and were attached to a small wire ring, which rested on it in the middle.

To perform the experiment, the solid, which was usually made of wax or boxwood, was placed on small pins projecting from the side of the trough. The threads were then stretched across the basin to the opposite side, and led from there up to a ring close under the celluloid bridge. From this point they passed along the underside of the bridge to the centre, where they passed up through the central hole.

The tank was then set rotating, and, when the whole system had attained a constant speed, a steel pricker was inserted into the wire ring, and raised approximately vertically. The sketch (text-fig. 2) shows the apparatus in action. In the case of the cylinder two threads were used to ensure that the axis should remain vertical. This is the case shown in the sketch. With the sphere, only one thread was used.

FIG 2.

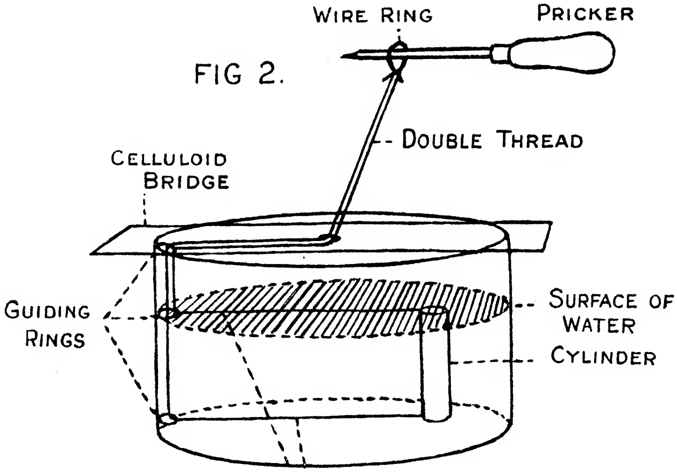

FIG. 2.—Rotating Tank arranged for Experiment with Cylinder.

The results in the two cases are shown in the photographs (Plate 2, figs. 1 and 2). In fig. 1 the cylinder is shown at the middle point of its path. Since its axis is nearly vertical it is seen end-on, and appears therefore as a black circle in the photograph. In the particular experiment shown in fig. 1, the axis of the cylinder had accidentally got tilted very slightly away from the vertical, so that both the upper and lower threads show as two distinct threads. The pricker and wire ring are seen at the bottom of the photograph; they are naturally rather out of focus. The lines in the photograph which are not straight are threads used for drawing the solids in other experiments. They have nothing to do with the present experiment. It will be seen that the predicted result is verified. The cylinder is moving practically straight across under the middle line of the celluloid bridge.

Fig. 2 shows the result in the case of the sphere. In this case it will be seen that the sphere is being deflected through a large angle, so that it is under the edge of the bridge instead of being under the middle, as it was in the case of the cylinder. It is not moving in the direction in which the thread is pulling it, but is being deflected to the right. In this case the liquid was rotating in the direction opposite to that of the hands of a clock. Again, this is the result which was predicted by theory.

Motion of Vortex Rings in a Rotating Fluid.

The difference between two- and three-dimensional fluid motion which has been discussed must apply to all fluid motions. It must therefore apply to the propagation of a vortex ring, as well as to the motions of a sphere and cylinder.

It is impossible, apparently, to produce a two-dimensional analogue of a vortex ring. If such a thing could be produced, it should, according to this theory, propagate itself in a straight line through a fluid, whether the fluid is rotating or not. In other words, since a rotation of the whole system should make no difference to any two-dimensional flow, it should make no difference to the two-dimensional analogue of a vortex ring.

The flow in a vortex ring, however, is three-dimensional, and a rotation of the whole system should affect it. For reasons explained in the paper already referred to,* it was anticipated on theoretical grounds that if a vortex ring can be propagated in a rotating fluid, it will not move in a straight line relative to the rotating system, but will move in a circle in the *opposite direction* to that in which the whole system is rotating.

In fig. 3 is shown a photograph of a vortex ring which has been projected from a small vortex box immersed in the water.† The basin was not rotating in this experiment. It will be seen that the ring, which is being projected along a diameter of the basin, moves in a straight line, as was to be expected. A short wire was attached to the vortex box, so as to point in the direction in which the ring was aimed. This will be seen in the photograph (fig. 3).

Fig. 4 is a photograph of a vortex ring projected when the whole system is rotating. In this photograph, the predicted curved path traversed by the ring is shown up clearly by the track of coloured fluid left behind during its flight.

The jet which drives the apparatus can be seen on the right-hand side of the photograph. It will be noticed that the basin is being driven in the counter-clockwise direction, and that the vortex ring is going round its circle in a clockwise direction, as predicted theoretically.

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* *Loc. cit.*, p. 107.

† The liquid eventually used was not pure water, but an acid solution of ferrous sulphate. The rings consisted of a solution of permanganate of potash made up to the same density as the ferrous sulphate solution by mixing with a heavy neutral salt. The rings then dissolved after they had broken up and a large number could be projected without discolouring the solution. For the suggestion to use this solution I am indebted to Dr. A. A. Robb.

One point which the experiments bring out is that the direction of the axis of the ring appears to be fixed in space, so that the ring would go round in a circle once during each revolution of the system. It would therefore be possible to consider the motion as being a steady motion relative to axes whose directions are fixed in space, but whose origin moves and is situated at the centre of the path of the vortex ring.

Slow Motions in a Rotating Fluid.

The investigations described above naturally led to inquiries as to whether rotating liquids possess any other properties which can be predicted from hydrodynamical theory. The following striking peculiarity of rotating fluids was discovered in the course of this work. If any small motion is communicated to a fluid which is initially rotating steadily like a solid body, the resulting flow must be two-dimensional, though small oscillations about this state of slow motion are possible.* This may be proved as follows:—

Let u, v, w be the components of velocity of any particle of fluid relative to a system which is rotating uniformly with velocity ω about the axis of z.

The circulation round any circuit in the fluid is

$$\mathrm{I} = \int \{(u-\omega y)\,dx + (v+\omega x)\,dy + w\,dz\},$$

the integral being taken round the circuit.

This may be divided into two parts

$$(a) \qquad \mathrm{I}' = \int \{u\,dx + v\,dy + w\,dz\}$$

which may be called the circulation due to the relative motion, and

$$(b) \qquad \int (-\omega y\,dx + \omega x\,dy)$$

which can be expressed in the form $2\omega\mathrm{A}$, where A is the area of the projection of the circuit on a plane perpendicular to the axis of rotation.

In a non-viscous fluid the circulation round any circuit which always consists of the same ring of particles, is constant. Hence it will be seen that $\mathrm{I} = \mathrm{I}' + 2\omega\mathrm{A}$ is constant.

Evidently if the motion relative to the rotating system is small this means that the variations in A during the whole motion are small. The liquid must therefore move in such a way that the area of the projection of any ring of particles on a plane perpendicular to the axis of rotation is nearly constant.

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* This is practically the same thing as the fact previously noted by Proudman, that small steady motions of a rotating fluid are two-dimensional.

Let us now enquire how this geometrical condition may be expected to reveal itself during the motion. First, we shall see what types of motion are possible in a fluid for which the areas of the projections on a given plane of all possible circuits of particles remain constant during the motion. This condition may be expressed mathematically by writing down an expression for the rate at which the area of the projection increases and equating it to 0. In this way it is found that

$$\int (vdx - udy) = 0,$$

the integral being taken round the circuit.

This expression may be transformed by Stokes' Theorem into the form

$$\iint \left\{ l\frac{\partial u}{\partial z} + m\frac{\partial v}{\partial z} - n\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \right\} dS = 0,$$

where dS is an element of surface of any surface which is bounded by the circuit in question and l, m, n are the direction cosines of the normal to that surface.

Since this relation holds for all possible circuits,

$$\partial u/\partial z = 0, \quad \partial v/\partial z = 0, \quad \text{and} \quad \partial u/\partial x + \partial v/\partial y = 0. \tag{1}$$

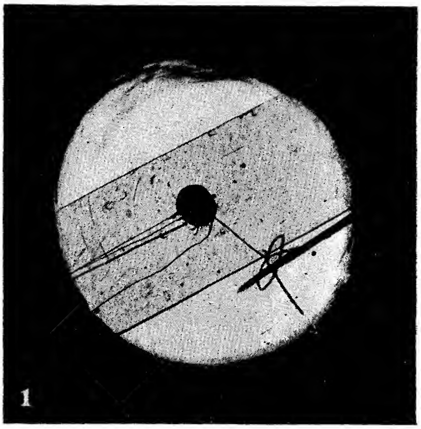
Hence, since the fluid is incompressible,

$$\partial w/\partial z = 0. \tag{2}$$

The conditions (1) show that any two particles which are originally in a line perpendicular to the given plane, will always remain in a line perpendicular to it. The condition (2) shows that they also remain at a constant distance apart throughout the motion.

If therefore any small motion be communicated to a rotating fluid the resulting motion of the fluid must be one in which any two particles originally in a line parallel to the axis of rotation remain so, except for possible small oscillations about that position. This property of rotating fluids is found to be true experimentally. It can be demonstrated in a very striking way by means of the apparatus described in the first part of this paper. The liquid is first made to rotate steadily as a solid body. A small motion is then communicated to it, and a few drops of coloured liquid are inserted. However carefully these drops are inserted the volume occupied by coloured water necessarily measures at least half a centimetre in every direction.

The slow movement of the fluid then draws this coloured portion of the fluid out into sheets. These sheets remain always parallel to the axis of rotation. They go on spreading almost indefinitely till they may perhaps be twenty or thirty times as long as the diameter of the basin, their thickness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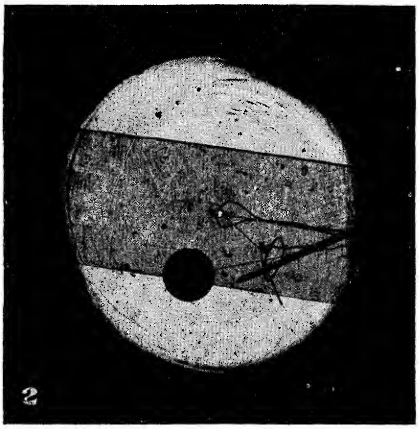
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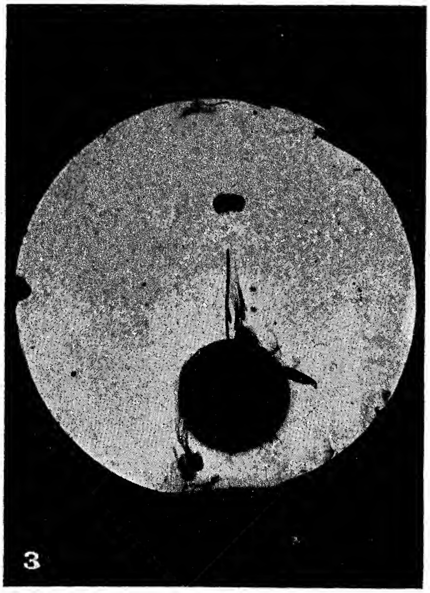
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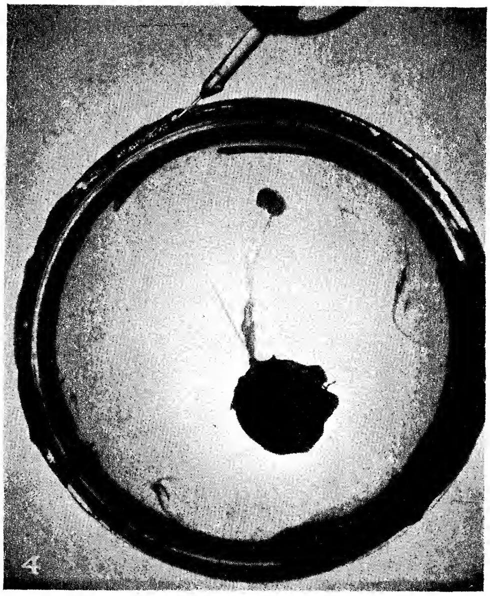
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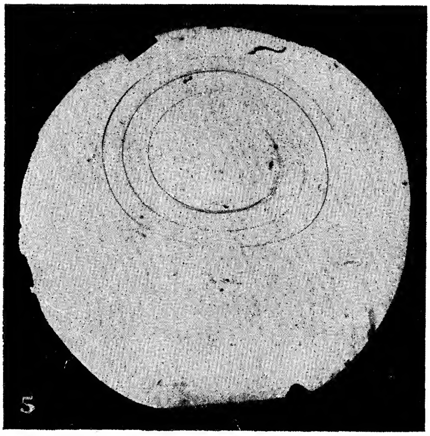
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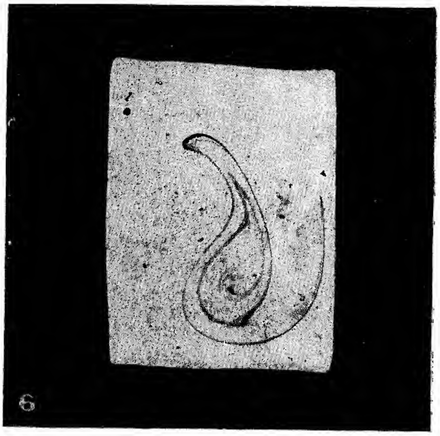
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decreases correspondingly till they are only a small fraction of a millimetre thick.

The accuracy with which they remain parallel to the axis of rotation is quite extraordinary. After the motion has been going on for some time it is only possible to see that the colouring matter is not uniformly diffused through the liquid by placing one's eye on, or near, the axis of rotation. The portion of the fluid which is passing immediately beneath the eye then appears to be filled with fine lines which are, of course, in reality, thin sheets seen edgewise.

The photographs shown in figs. 5 and 6 were taken by a camera placed accurately on the axis of rotation of the basin. It will be seen that the lines are extremely fine. The sheets into which the originally diffuse patch of coloured liquid are drawn are therefore extremely thin, and are moreover accurately parallel to the axis of rotation. It appears therefore that in this case also the theoretical prediction is completely verified by experiments with real fluids.

In fig. 5 the small motion was communicated to the fluid by changing the speed of the rotating basin temporarily. Fig. 6 shows an experiment in which a rectangular boundary was placed in the rotating basin in order to alter the effect produced by a change of speed, and so produce a different pattern.

In a future paper the author hopes to discuss what happens in the case when the boundaries of the fluid move slowly in such a way that three-dimensional motion must take place.

DESCRIPTION OF PLATE.

Fig. 1.—Cylinder passing under centre of tank, seen from point above the tank and on the axis of rotation.

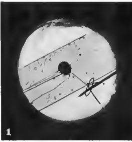
Fig. 2.—Sphere being deflected, seen from the same point as the cylinder shown in fig. 1.

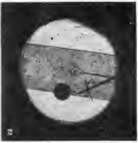
Fig. 3.—Vortex ring being projected in a non-rotating tank. The large black disc is the vortex box, seen end-on. The wire seen projecting from the box indicates the direction of projection of the ring. The ring will be seen clear of the end of the wire.

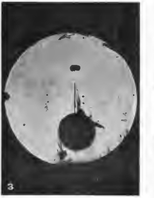
Fig. 4.—Vortex ring projected in rotating tank. The curved path is shown by the trail of coloured liquid left by the ring in its flight. The jet which drives the inner basin is seen at the top of the photograph.

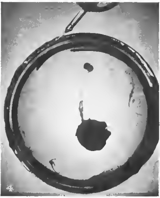
Fig. 5.—Sheet of coloured liquid seen from a point on the axis of rotation in a rotating liquid.

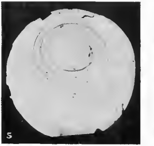
Fig. 6.—Another sheet of coloured liquid. In this case the liquid is contained in a rectangular boundary.

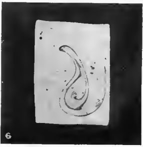












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